Question	Scheme	Marks	AOs
1(a)	$h = 0.5$	B1	1.1b
	$A \approx \frac{1}{2}\times\frac{1}{2}\{0.4805+1.9218+2(0.8396+1.2069+1.5694)\}$	M1	1.1b
	$= 2.41$	A1	1.1b
		(3)	
(b)	$\int(\ln x)^2\ \mathrm{d}x = x(\ln x)^2 - \int x\times\frac{2\ln x}{x}\ \mathrm{d}x$	M1 A1	3.1a 1.1b
	$= x(\ln x)^2 - 2\int \ln x\ \mathrm{d}x = x(\ln x)^2 - 2\left(x\ln x - \int \mathrm{d}x\right)$ $= x(\ln x)^2 - 2\int \ln x\ \mathrm{d}x = x(\ln x)^2 - 2x\ln x + 2x$	dM1	2.1
	$\int_2^4 (\ln x)^2\ \mathrm{d}x = \left[x(\ln x)^2 - 2x\ln x + 2x\right]_2^4$ $= 4(\ln 4)^2 - 2\times 4\ln 4 + 2\times 4 - \left(2(\ln 2)^2 - 2\times 2\ln 2 + 2\times 2\right)$ $= 4(2\ln 2)^2 - 16\ln 2 + 8 - 2(\ln 2)^2 + 4\ln 2 - 4$	ddM1	2.1
	$= 14(\ln 2)^2 - 12\ln 2 + 4$	A1	1.1b
		(5)	
		(8 marks)	

Notes

(a)

B1: Correct strip width. May be implied by $\frac{1}{2}\times\frac{1}{2}\{....\}$ or $\frac{1}{4}\times\{....\}$

M1: Correct application of the trapezium rule.

Look for $\frac{1}{2}\times "h"\{0.4805+1.9218+2(0.8396+1.2069+1.5694)\}$ condoning slips in the digits.

The bracketing must be correct but it is implied by awrt 2.41

A1: 2.41 only. This is not awrt

(b)

M1: Attempts parts the correct way round to achieve $\alpha x(\ln x)^2 - \beta\int \ln x\ \mathrm{d}x$ o.e.

May be unsimplified (see scheme). Watch for candidates who know or learn $\int \ln x\ \mathrm{d}x = x\ln x - x$

who may write $\int(\ln x)^2\ \mathrm{d}x = \int(\ln x)(\ln x)\ \mathrm{d}x = \ln x(x\ln x - x) - \int\frac{x\ln x - x}{x}\ \mathrm{d}x$

A1: Correct expression which may be unsimplified

dM1: Attempts parts again to (only condone coefficient errors) to achieve $\alpha x(\ln x)^2 - \beta x\ln x \pm \gamma x$ o.e.

ddM1: Applies the limits 4 and 2 to an expression of the form $\pm\alpha x(\ln x)^2 \pm \beta x\ln x \pm \gamma x$, subtracts and applies $\ln 4 = 2\ln 2$ at least once. Both M's must have been awarded

A1: Correct answer

....

It is possible to do $\int(\ln x)^2\ \mathrm{d}x$ via a substitution $u = \ln x$ but it is very similar.

M1 A1, dM1: $\int u^2 \mathrm{e}^u\ \mathrm{d}u = u^2\mathrm{e}^u - \int 2u\,\mathrm{e}^u\ \mathrm{d}u, = u^2\mathrm{e}^u - 2u\mathrm{e}^u \pm 2\mathrm{e}^u$

ddM1: Applies appropriate limits and uses $\ln 4 = 2\ln 2$ at least once to an expression of the form $u^2\mathrm{e}^u - \beta u\mathrm{e}^u \pm \gamma\mathrm{e}^u$ Both M's must have been awarded

....

Question	Scheme	Marks	AOs
2(a)	States or uses $h = 1.5$	B1	1.1a
	Full attempt at the trapezium rule $= \frac{\dots}{2}\{1.63 + 2.63 + 2\times(2 + 2.26 + 2.46)\}$	M1	1.1b
	= awrt 13.3 or $\frac{531}{40}$	A1	1.1b
		(3)	
(b)(i)	$\int_3^9 \log_3(2x)^{10}\,\mathrm{d}x = 10\times\text{"}13.3\text{"} = \text{awrt}\,133$ or e.g. $\frac{531}{4}$	B1ft	2.2a
(ii)	$\int_3^9 \log_3 18x\,\mathrm{d}x = \int_3^9 \log_3(9\times 2x)\,\mathrm{d}x = \int_3^9 2 + \log_3 2x\,\mathrm{d}x$ $= [2x]_3^9 + \int_3^9 \log_3 2x\,\mathrm{d}x = 18 - 6 + \int_3^9 \log_3 2x\,\mathrm{d}x = \dots$	M1	3.1a
	Awrt 25.3 or $\frac{1011}{40}$	A1ft	1.1b
		(3)	
		(6 marks)	

Notes:

(a)

B1: States or uses $h = 1.5$

M1: A full attempt at the trapezium rule.

Look for $\frac{\text{their } h}{2}\{1.63 + 2.63 + 2\times(2 + 2.26 + 2.46)\}$ but condone copying slips

Note that $\frac{\text{their } h}{2}1.63 + 2.63 + 2\times(2 + 2.26 + 2.46)$ scores M0 unless the missing brackets are recovered or implied by their answer. You may need to check.

Allow this mark if they add the areas of individual trapezia e.g.

$\frac{\text{their } h}{2}\{1.63 + 2\} + \frac{\text{their } h}{2}\{2 + 2.26\} + \frac{\text{their } h}{2}\{2.26 + 2.46\} + \frac{\text{their } h}{2}\{2.46 + 2.63\}$

Condone copying slips but must be a complete method using all the trapezia.

A1: awrt 13.3 (Note full accuracy is 13.275) or exact equivalent.

Note that the calculator answer is 13.324 so you must see correct working to award awrt 13.3

Use of $h = -1.5$ leading to a negative area can score B1M1A0 but allow full marks if then stated as positive.

(b)(i)

B1ft: Deduces that $\int_3^9 \log_3(2x)^{10}\,\mathrm{d}x = 10\times\text{"}13.3\text{"} = \text{awrt } 133$

FT on their 13.3 look for 3sf accuracy but follow through on e.g. their rounded answer to part (a) so if 13 was their answer to part (a) then allow 130 here **following a correct method**.

A correct method must be seen here but a minimum is e.g. 10 × “13.3” = “133”

Note that $\int_3^9 \log_3(2x)^{10}\,\mathrm{d}x = 133.2414316\dots$ so a correct method must be seen to award marks.

Attempts to apply the trapezium rule again in any way score M0 as the instruction in the question was to use the <u>answer</u> to part (a).

(b)(ii)

M1: Shows correct log work to relate the given question to part (a)

Must reach as far as e.g. $[2x]_3^9 + \int_3^9 \log_3 2x \, dx = ...$ with correct use of limits on $[2x]_3^9$ which may be implied or equivalent work e.g. finds the area of the rectangle as 2×6

A1ft: **Correct working** followed by awrt 25.3 but ft on their 13.3 so allow for 12 + their answer to part (a) **following correct work** as shown.

Note that $\int_3^9 \log_3 18x \, dx = 25.32414...$ **so a correct method must be seen to award marks**.

Some examples of an acceptable method are:

$$\int_3^9 \log_3 18x \, dx = \int_3^9 \log_3 (9 \times 2x) \, dx = \int_3^9 2 + \log_3 2x \, dx = 6 \times 2 + "13.3" = 25.3$$

$$\int_3^9 \log_3 18x \, dx = \int_3^9 \log_3 (9 \times 2x) \, dx = \int_3^9 2 + \log_3 2x \, dx = 12 + "13.3" = 25.3$$

$$\int_3^9 \log_3 18x \, dx = \int_3^9 \log_3 (9 \times 2x) \, dx = \int_3^9 2 + \log_3 2x \, dx = [2x]_3^9 + \int_3^9 \log_3 2x \, dx = 25.3$$

BUT just $12 + "13.3" = 25.3$ **scores M0**

Attempts to apply the trapezium rule again in any way score M0 as the instruction in the question was to use the answer to part (a).

Question	Scheme	Marks	AOs
3(a)	$h=0.2$	B1	1.1b
	$\frac{1}{2}\times"0.2"\times\{a+13.5+2(16.8+b+20.2+18.7)\}=17.59$	M1	1.1b
	e.g. $\Rightarrow a+13.5+2b+111.4=175.9\Rightarrow a+2b=51$*	A1*	2.1
		(3)	
(b)	$a+16.8+b+20.2+18.7+13.5=97.2\Rightarrow a+b=28\Rightarrow a=...$ (or $b=...$)	M1	3.1a
	$a=5$ or $b=23$	A1	1.1b
	$a=5$ and $b=23$	A1	1.1b
		(3)	
			(6 marks)

Notes

(a)

B1: States or uses $h=0.2$ o.e.

M1: Forms the equation $\frac{1}{2}\times"0.2"\times\{a+13.5+2(16.8+b+20.2+18.7)\}=17.59$ o.e. but condone copying slips. They may have added some of the *y* values together so as a minimum accept e.g. $"0.1"\times\{a+13.5+2(55.7+b)\}=17.59$

Condone invisible brackets as long as they are recovered or implied in further work before achieving the given answer. Condone the use of $\approx$ for this mark.
Allow this mark if they add the areas of individual trapezia e.g.
$\frac{\text{their } h}{2}\{a+16.8\}+\frac{\text{their } h}{2}\{16.8+b\}+\frac{\text{their } h}{2}\{b+20.2\}+\frac{\text{their } h}{2}\{20.2+18.7\}+\frac{\text{their } h}{2}\{18.7+13.5\}$
Condone copying slips but it must be a complete method using all the trapezia. *h* must be numerical but condone $h=1$

A1*: A rigorous argument leading to $a+2b=51$ from correct working and no errors seen including brackets, although do not penalise a missing trailing bracket at the end e.g.
$\frac{1}{2}\times"0.2"\times\{a+13.5+2(16.8+b+20.2+18.7)=17.59\Rightarrow...\Rightarrow a+2b=51$ could score B1M1A1but
$\frac{1}{2}\times"0.2"\times a+13.5+2(16.8+b+20.2+18.7)=17.59\Rightarrow...\Rightarrow a+2b=51$ could score max B1M1A0
provided later work implied correct brackets.
Both sets of brackets must be dealt with correctly before proceeding to the final answer such that
e.g. $...\Rightarrow a+2b+124.9=175.9\Rightarrow a+2b=51$ is M1A1*
$...\Rightarrow a+13.5+33.6+2b+40.4+37.4=175.9\Rightarrow a+2b=51$ is M1A1*
$...\Rightarrow 0.1a+1.35+3.36+0.2b+4.04+3.74=17.59\Rightarrow a+2b=51$ is M1A1*
Note that $a+2b\approx 51$ as the **final** answer is A0*

(b)

M1: Attempts to form the equation $a+16.8+b+20.2+18.7+13.5=97.2$, condoning copying errors, (may just be stated as e.g. $a+b=28$ o.e.) and attempts to solve their equation simultaneously with the given equation (or condone their equation from part (a)). Do not be too concerned with the process here as calculators may be used. Score if values for *a* or *b* are reached from a pair of simultaneous equations.

A1: for $a=5$ or $b=23$

A1: for both $a=5$ and $b=23$